Structural Response to Segmented Nonstationary Random Excitation

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This paper presents a procedure for calculating the mean square response of a single-degree-of-freedom linear structural system to a particular type of nonstationary random forcing function. The forcing function, herein referred to as segmented nonstationary, is a stochastic process generated by adding a series of covariance stationary zero-mean random forcing functions which are each shaped by deterministic functions of time that do not overlap in the time domain. The system's mean square response is formulated in terms of the segments' time-dependent frequency response functions and the forcing functions' stationary spectral density functions. The results are specialized to consider the response to forcing functions time-modulated by rectangular pulses.

Nomenclature

$e_r(t)$ = deterministic modulating function of rth segment
$f_r(t)$ = covariance stationary forcing function of rth segment
$H(\omega)$ = system frequency response function relating displacement response and excitation
$I_r(t, \omega)$ = time-dependent frequency response function
m = mass
$q(t)$ = oscillator displacement
$Q(t)$ = input excitation
$R_{f_r f_s}(t_1, t_2)$ = cross-correlation function relating $f_r(t_1)$ and $f_s(t_2)$
$S_{f_r f_s}(\omega)$ = cross-spectral density function between forcing function $f_r(t)$ and $f_s(t)$
ω_0 = system undamped natural frequency
ζ = critical damping ratio

Introduction

A FUNDAMENTAL difference between a stationary and a nonstationary random forcing function is that both the frequency content and the intensity are time-invariant in the former and time-variant in the latter. The reasons for these variations in frequency content and intensity are usually attributed to some physical phenomenon, i.e., the firing of different rocket stages, the turbulence encountered by aircraft during flight, the terrain traversed by transport vehicles, or the various stages of earthquake ground motion. Often it is observed that the forcing functions exciting the structural system possess time-variant intensity and frequency content. In many cases the frequency content does change with time, but in such a manner that there are intervals or segments of time where it can be assumed to be stationary. It is the goal of this paper to develop a mathematical description of these forcing functions, and also to help provide insight into the response of single-degree-of-freedom systems to this excitation.

A number of studies have been made which seek to describe the dynamic response of simple structural systems to stochastic forcing functions. Most of the early work was limited to stationary processes because of their mathematical simplicity. More recently, models for nonstationary processes have been developed. These include: 1) a finite sum of time-modulated harmonics with random phasing,[1] and 2) filtered Poisson processes[2,3] time-modulated stationary processes.[4–10]

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Of interest in this paper is the dynamic response of a single-degree-of-freedom linear structural system, initially, at rest, to a segmented nonstationary random forcing function. The forcing function takes the form

$$Q(t) = \sum_{r=1}^{n} e_r(t) f_r(t) \tag{1}$$

It is a stochastic process generated by adding a series of n covariance stationary zero-mean random forcing functions $f_r(t)$ that are each shaped or modulated by deterministic functions of time $e_r(t)$ that do not overlap in the time domain.

Several investigators have studied the dynamic response of a single-degree-of-freedom linear structural system to one stationary random forcing function $f_1(t)$ time-modulated by a single deterministic shape function $e_1(t)$. Included in these studies were white noise processes time-modulated by the step, rectangular, and half-sine functions,[4,5] and filtered white noise processes time-modulated by step, rectangular, increasing-decreasing and decreasing exponential functions.[6,7]

The main advantage of the approach presented in this paper over those mentioned previously is that the analyst can vary both the intensity and the frequency content of the input excitation with time. This is due to the fact that the time domain is segmented, and the intensity and frequency content of each segment is allowed to be unique.

As an example, consider the ensemble of hypothetical records (NSF in number) of a nonstationary input excitation shown in Fig. 1a. If the records are stationary in each segment, the deterministic shape function $e_r(t)$ for the rth segment takes the form of a rectangular pulse, i.e.,

$$e_r(t) = U(t - t_{r-1}) - U(t - t_r) \tag{2}$$

as shown in Fig. 1b. Furthermore, if these shaping functions do not overlap in the time domain, estimates of the stationary direct-spectral density $S_{f_r f_r}(\omega)$ within the rth segment, and the

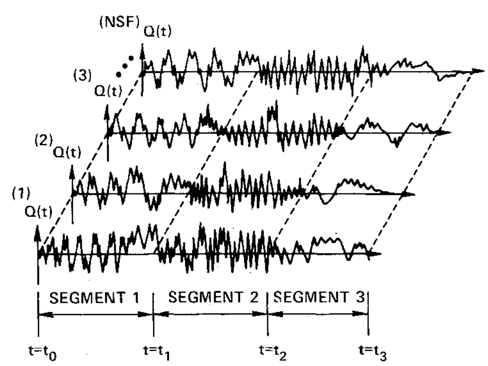


Fig. 1a Ensemble of records of a single nonstationary input excitation.

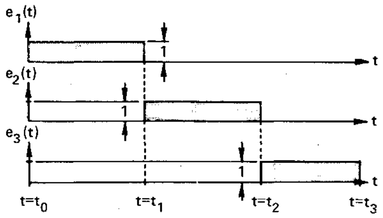


Fig. 1b Shaping functions for rectangular time pulse modulation.

stationary cross-spectral density $S_{f_r f_s}(\omega)$ between the rth and sth segments, can be obtained readily using either autocorrelation or Fourier transform operations on the data in these segments. It should be pointed out, however, that the statistical confidence of these estimates may be quite low if the time duration of any segment is short. Even under these conditions, it is sometimes possible to estimate accurately the stationary direct and cross-spectral densities.

Consider, for example, the nonstationary acoustical excitation associated with the launching of multistage space vehicles. This excitation has, in general, a constant or slowly varying frequency content during each stage of launch. For this case, one can assume the excitation for each stage is a stationary stochastic process. Experimental tests can be designed to simulate the excitation for each stage; however, the time duration of each of these tests must be long enough to obtain satisfactory estimates of its stationary statistical properties. The time history of the excitation for each stage, recorded during these tests, may then be used to calculate estimates of each stage's direct-spectral density, as well as the cross-spectral densities between stages. Such a procedure can similarly be carried out for vehicle transport and other structural problems.

In this paper, the mean square response $E[q^2(t)]$ is derived in the form

$$E[q^2(t)] = \sum_{r=1}^{n} \sum_{s=1}^{n} \int_{-\infty}^{\infty} S_{f_r f_s}(\omega) I_r(t,\omega) I_s^*(t,\omega)\, d\omega \qquad (3)$$

where $S_{f_r f_s}(\omega)$ is the two-sided cross-spectral density function between forcing functions, $f_r(t)$ and $f_s(t)$, and $I_r(t,\omega)$ are the time-dependent frequency response functions for the corresponding shape functions, $e_r(t)$ and $e_s(t)$. The behavior of the product $I_r(t,\omega)I_s^*(t,\omega)$ for the special case of rectangular time-pulse modulation is studied in both the frequency and time domains as a function of pulse duration and structure damping. The errors introduced by neglecting the contribution to the mean square response of terms containing the cross-product $I_r(t,\omega)I_s^*(t,\omega)$ are discussed. The mean square response to stepped narrow-band random excitation is treated as an example.

Response to Segmented Nonstationary Random Excitation

The differential equation of motion for a single-degree-of-freedom linear structural system can be written as

$$\ddot{q}(t) + 2\zeta\omega_0\dot{q}(t) + \omega_0^2 q(t) = Q(t)/m \qquad (4)$$

where m is the mass, ω_0 is the undamped natural frequency, ζ is the critical damping ratio, $q(t)$ is the system response, and $Q(t)$ is the input excitation.

The system's nonstationary mean square response can be expressed in the form[11]

$$E[q^2(t)] = \int_{-\infty}^{\infty} S_{QQ}(\omega_1,\omega_2) H(\omega_1) H^*(\omega_2) e^{i(\omega_1-\omega_2)t}\, d\omega_1\, d\omega_2 \qquad (5)$$

where $S_{QQ}(\omega_1,\omega_2)$ is the generalized spectral density function for the input excitation and $H(\omega)$ is the frequency response function relating system response and input excitation.

When the input excitation is segmented nonstationary, it may be written in the form

$$Q(t) = \sum_{r=1}^{n} e_r(t) f_r(t) \qquad (6)$$

where $e_r(t)$ is any deterministic shaping function and $f_r(t)$ is a covariance stationary zero mean random forcing function. The autocorrelation function $R_{QQ}(t_1,t_2)$ of this input excitation takes the form

$$R_{QQ}(t_1,t_2) = \sum_{r=1}^{n} \sum_{s=1}^{n} e_r(t_1) e_s(t_2) R_{f_r f_s}(t_1,t_2) \qquad (7)$$

since $e_r(t)$ and $e_s(t)$ are deterministic. Noting that $R_{QQ}(t_1,t_2)$ and $S_{QQ}(\omega_1,\omega_2)$ are related by the expression

$$S_{QQ}(\omega_1,\omega_2) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} R_{QQ}(t_1,t_2) e^{-i(\omega_1 t_1 - \omega_2 t_2)}\, dt_1\, dt_2 \qquad (8)$$

one may then write

$$S_{QQ}(\omega_1,\omega_2) = \frac{1}{4\pi^2} \sum_{r=1}^{n} \sum_{s=1}^{n} \int_{-\infty}^{\infty} e_r(t_1) e_s(t_2) R_{f_r f_s}(t_1,t_2) e^{-i(\omega_1 t_1 - \omega_2 t_2)}\, dt_1\, dt_2 \qquad (9)$$

The cross-correlation function $R_{f_r f_s}(t_1,t_2)$ may be recast in the form $R_{f_r f_s}(t_1 - t_2)$, since both $f_r(t)$ and $f_s(t)$ are covariance stationary. Now, $R_{f_r f_s}(t_1 - t_2)$ and the two-sided stationary cross-spectral density function $S_{f_r f_s}(\omega)$, are related by the expression

$$R_{f_r f_s}(t_1 - t_2) = \int_{-\infty}^{\infty} S_{f_r f_s}(\omega) e^{i\omega(t_1 - t_2)}\, d\omega \qquad (10)$$

so Eq. (9) may be written in the alternate form

$$S_{QQ}(\omega_1,\omega_2) = \sum_{r=1}^{n} \sum_{s=1}^{n} \int_{-\infty}^{\infty} S_{f_r f_s}(\omega) A_r(\omega - \omega_1) A_s^*(\omega - \omega_2)\, d\omega \qquad (11)$$

where $A_r(\omega - \omega_1)$ is defined as

$$A_r(\omega - \omega_1) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} e_r(t_1) e^{i(\omega - \omega_1)t_1}\, dt_1 \qquad (12)$$

and $A_s^*(\omega - \omega_2)$ is the complex conjugate

$$A_s^*(\omega - \omega_2) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} e_s(t_s) e^{-i(\omega - \omega_2)t_2}\, dt_2 \qquad (13)$$

Substitution of Eq. (11) into Eq. (5) gives, after rearrangement

$$E[q^2(t)] = \sum_{r=1}^{n} \sum_{s=1}^{n} \int_{-\infty}^{\infty} S_{f_r f_s}(\omega) I_r(t,\omega) I_s^*(t,\omega)\, d\omega \qquad (14)$$

where $I_r(t,\omega)$ is called the time-dependent frequency response function, which is defined as

$$I_r(t,\omega) \equiv \int_{-\infty}^{\infty} H(\omega_1) A_r(\omega - \omega_1) e^{i\omega_1 t}\, d\omega_1 \qquad (15)$$

and $I_s^*(t,\omega)$ is the corresponding complex conjugate

$$I_s^*(t,\omega) \equiv \int_{-\infty}^{\infty} H^*(\omega_2) A_s^*(\omega - \omega_2) e^{-i\omega_2 t}\, d\omega_2 \qquad (16)$$

Response to Forcing Functions Time-Modulated by Rectangular Pulses

Equation (14) shows that the mean square response is a function of the forcing functions' direct and cross-spectral density functions and the product of the corresponding time-dependent frequency response functions. In this section the time-dependent frequency response function $I_r(t,\omega)$ will be evaluated for the special case of rectangular time-pulse modulation. The behavior of the product $I_r(t,\omega)I_s^*(t,\omega)$ will be studied in both the time and frequency domains as a function of pulse duration and structure damping in order to gain general insight into the nature of the system's response.

For rectangular time-pulse modulation, the shape function for the rth segment is given by Eq. (2), and Eq. (12) becomes

$$A_r(\omega - \omega_1) = \frac{1}{2\pi} \int_{t_{r-1}}^{t_r} e^{i(\omega - \omega_1)t_1}\, dt_1$$

$$= \left(\frac{1}{2\pi i}\right) \sum_{r_0=r-1}^{r} (-1)^{r-r_0} e^{\frac{i(\omega - \omega_1)t_{r_0}}{(\omega - \omega_1)}} \quad (17)$$

The time-dependent frequency response function, defined in Eq. (15), becomes

$$I_r(t, \omega) = \frac{1}{2\pi} \sum_{r_0=r-1}^{r} (-1)^{r-r_0} e^{i\omega t_r} \int_{-\infty}^{\infty} \frac{H(\omega_1) e^{i\omega_1(t-t_r)} d\omega_1}{(\omega - \omega_1)} \quad (18)$$

Evaluation of this expression, using the calculus of residues, yields[12,13]

$$I_r(t, \omega) = -H(\omega) \sum_{r_0=r-1}^{r} (-1)^{r-r_0} U(t - t_{r_0})$$
$$\left\{ e^{i\omega t} - e^{i\omega t_r} \left[C_0(t - t_{r_0}) + \frac{i\omega}{a_0} D_0(t - t_{r_0}) \right] \right\} \quad (19)$$

where

$$H(\omega) = 1/m(\omega_0^2 - \omega^2 + i2\zeta\omega_0\omega) \quad (20)$$

$$a_0 = \omega_0(1 - \zeta^2)^{1/2} = \text{damped natural frequency of vibration} \quad (21)$$

$$b_0 = \zeta\omega_0 \quad (22)$$

$$C_0(t) = e^{-b_0 t}[\cos a_0 t + (b_0/a_0) \sin a_0 t] \quad (23)$$

$$D_0(t) = e^{-b_0 t} \sin a_0 t \quad (24)$$

The behavior of the product $I_r(t, \omega)I_s^*(t, \omega)$ is illustrated in Figs. 2–5. The figures are presented in the dimensionless form $4\zeta^2 m^2 \omega_0^4 I_r(t/T_0, \omega/\omega_0) I_s^*(t/T_0, \omega/\omega_0)$ where $\frac{1}{4}(\zeta^2 m^2 \omega_0^4)$ is the peak stationary value of $I_r(t, \omega)I_s^*(t, \omega)$, and T_0 is the undamped natural period of the system.

Figures 2a and 3a illustrate the behavior of the product $|I_r(t, \omega)|^2$ as a function of frequency at various instants of time during its build-up and die-down phases, respectively. The accompanying figures, Figs. 2b and 3b, show the time history of $|I_r(t, \omega_0)|^2$ during these phases. Several observations can be drawn from these figures.

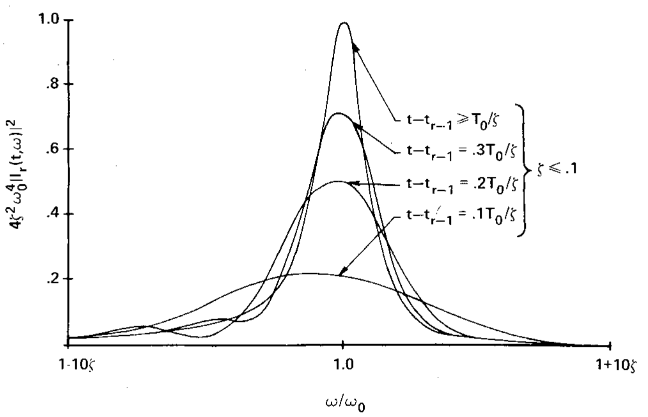


Fig. 2a Behavior of $|I_r(t,\omega)|^2$ as a function of frequency at various instants of time during the build-up phase.

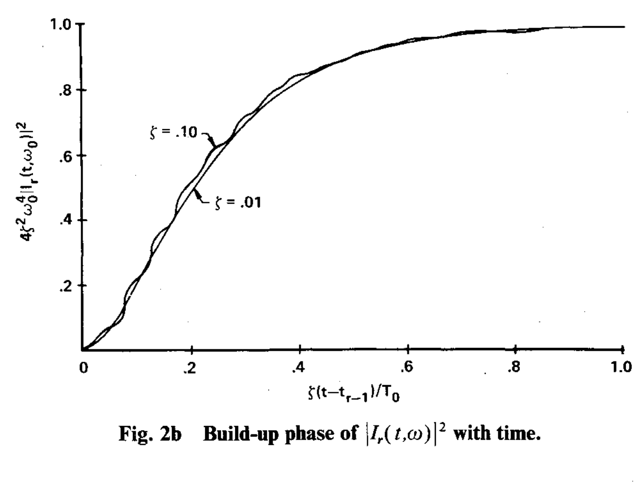


Fig. 2b Build-up phase of $|I_r(t,\omega)|^2$ with time.

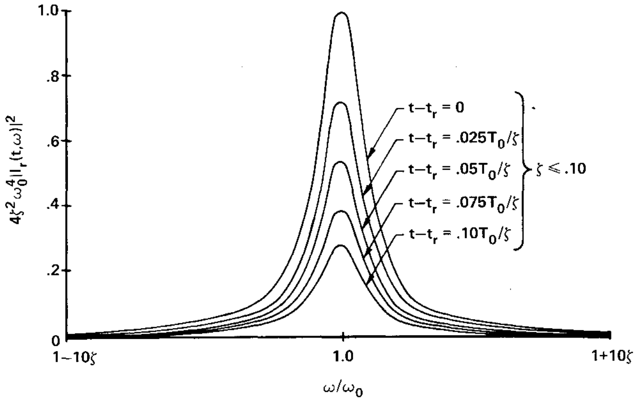


Fig. 3a Behavior of $|I_r(r,\omega)|^2$ as a function of frequency at various instants of time during the die-down phase.

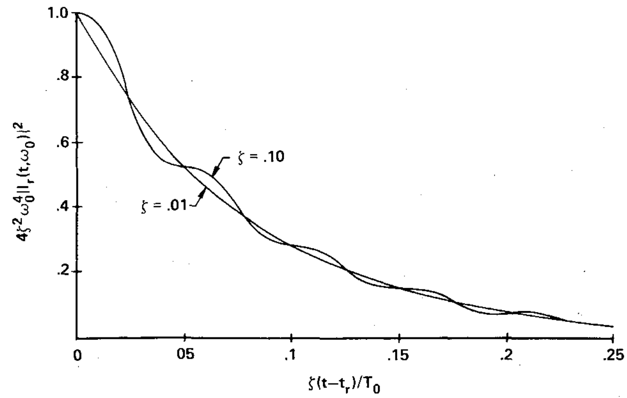


Fig. 3b Die-down phase of $|I_r(t,\omega)|^2$ with time.

1) Figures 2a and 2b indicate that $|I_r(t, \omega)|^2$ exhibits an oscillatory character of indeterminant frequency as it builds up to 99% of its stationary value $|H(\omega)|^2$ in T_0/ζ sec. 2) Figures 3a and 3b show that $|I_r(t, \omega)|^2$ has an oscillatory character with a frequency $2\omega_0$ as it dies down to 1% of its stationary value in $T_0/2\zeta$ sec. 3) Figure 2a also shows that the transient value of $|I_r(t, \omega)|^2$ may, at some point in time, exceed its stationary value except at the system's natural frequency. For example, at $\omega/\omega_0 = 1 - 5\zeta$ the value at $t - t_{r-1} = 0.1T_0/\zeta$ is twice the stationary value. This suggests that the transient response may significantly exceed the stationary response if the direct-spectral density for the rth segment is peaked at frequencies away from the system's natural frequency.

Figure 4a illustrates the behavior of $I_r(t, \omega)I_s^*(t, \omega)$ as a function of frequency at various instants of time in the transition region $t \geqq t_r$, immediately following the end of the rth time segment. The accompanying figure (Fig. 4b) shows the time history of $I_r(t, \omega_0)I_s^*(t, \omega_0)$ during this time period. The following observations can be drawn from these figures.

1) Figure 4a shows that the real and imaginary parts of

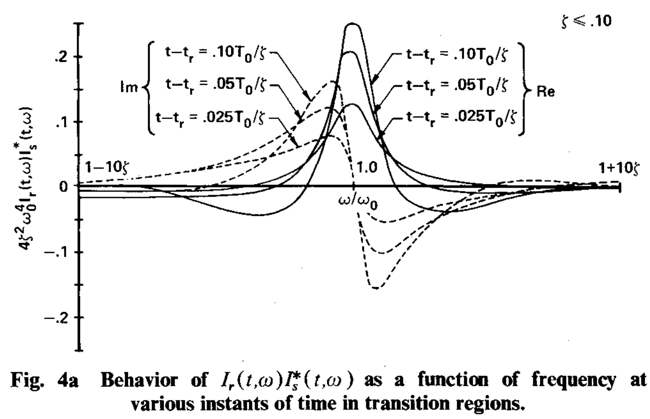


Fig. 4a Behavior of $I_r(t,\omega)I_s^*(t,\omega)$ as a function of frequency at various instants of time in transition regions.

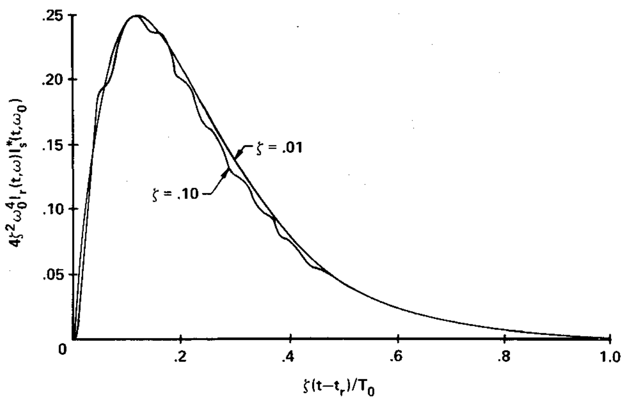


Fig. 4b Behavior of $I_r(t,\omega_0)I_s^*(t,\omega_0)$ in transition regions.

$I_r(t,\omega)I_s^*(t,\omega)$ are significant only in the region $\omega_0 \pm 10\zeta\omega_0$. The real part has a peak positive value at $\omega = \omega_0$ where the imaginary part is zero. The imaginary part has peak positive and negative values at ω_0^- and ω_0^+, respectively. 2) Figure 4b indicates that the peak modulus $I_r(t,\omega_0)I_s^*(t,\omega_0)$ attains a maximum value of 25% of the stationary peak value of $|I_r(t,\omega_0)|^2$ at $T_0/10\zeta$ sec into the second pulse, and dies down to 1% of its stationary value in T_0/ζ sec.

Plots of the relative contribution of both direct and cross-product terms in the transition region $t \geqq t_r$ are given in Fig. 5. This figure shows that at $(t - t_r) = 0.1T_0/\zeta$, the contribution of the sum of the two cross-products terms is equal to the sum of the two direct-product terms.

Errors generated by ignoring the contribution of terms containing the cross-product $I_r(t,\omega)I_s^*(t,\omega)$ can now be discussed in view of the observations drawn from these figures. Figure 5 shows that the contribution of the sum of the two cross-product terms may reach 50% of the total response when the direct-spectral densities in adjacent segments are peaked at the system's natural frequency. On the other hand, Fig. 4a suggests that the contribution of these cross-product terms may be small whenever the cross-spectral densities are nearly uniform or small over the frequency range $\omega_0 \pm 10\zeta\omega_0$. It should also be noted that the cross-spectral densities are zero whenever the frequency content of segments r and s does not overlap in view of the inequality

$$|S_{f_rf_s}(\omega)|^2 \leqq S_{f_rf_r}(\omega)S_{f_sf_s}(\omega) \tag{25}$$

Example: Response to Stepped-Narrowband Random Excitation

Consider the unifrom cantilever shown in Fig. 6a, excited by the stepped narrow-band base acceleration shown in Fig. 6b. The uniform cantilever has mass m_0 per unit length, length l, and flexural stiffness EI.

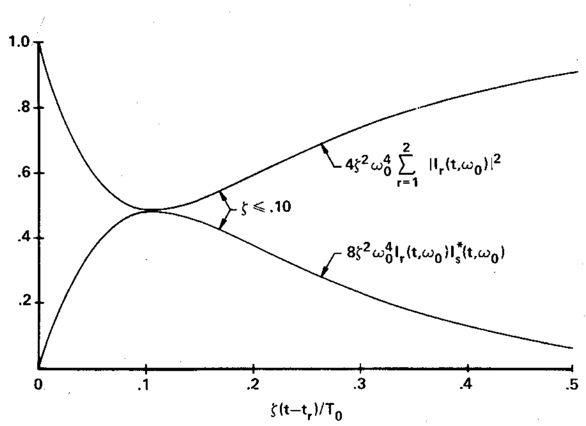


Fig. 5 Relative contribution of direct and cross-product terms in transition regions.

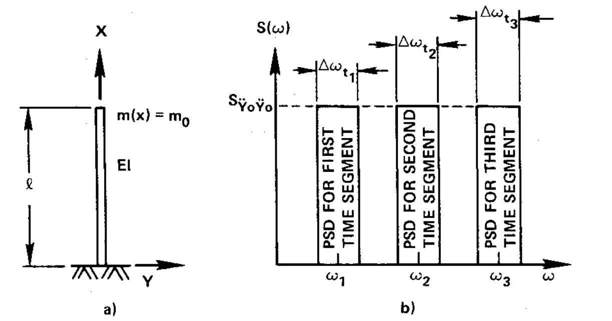


Fig. 6 Uniform cantilever excited by stepped-narrow-band base acceleration.

Assuming the response $y(x, t)$ at any point x at time t can be expressed as

$$y(x,t) = \Phi(x)q(t) \tag{26}$$

where

$$\Phi(x) = 1 - \cos(\pi x/2l) \tag{27}$$

the system mass m, stiffness k, and first undamped natural frequency become

$$m = \int_0^l m(x)\Phi^2(x)\,dx^2 = 0.22676\,m_0 l \tag{28}$$

$$k = \int_0^l EI(x)[\Phi(x)]^2\,dx = 0.03125\,\pi^4\frac{EI}{l^3} \tag{29}$$

$$\omega_0 = (k/m)^{1/2} = 0.37123\frac{\pi^2}{l^2}(EI/m_0)^{1/2} \tag{30}$$

The generalized force $Q(t)$ for inertial loading due to base acceleration excitation $y_0(t)$ is

$$Q(t) = \int_0^l \Phi(x)m(x)\ddot{y}_0(t)\,dx = 0.36338\,m_0 l y_0(t) \tag{31}$$

For the stepped narrow-band base acceleration excitation shown in Fig. 6b, obtained by passing white noise through a series of time-sequenced idealized narrow-band-pass filters, the deterministic shaping functions $e_r(t)$ are rectangular pulses taking the form defined by Eq. (2) and the direct spectral densities $S_{f_rf_r}(\omega)$ are specified as:

$$S_{f_rf_r}(\omega) = S_{f_rf_r}, \quad \omega_{f_r} - 0.5\Delta\omega_{f_r} \leqq \omega \leqq \omega_{f_r} + 0.5\Delta\omega_{f_r} \tag{32}$$
$$= 0, \quad \text{elsewhere}$$

where ω_{f_r} is the center frequency and $\Delta\omega_{f_r}$ is the bandwidth of the rth filter. If the frequency content of segments r and s does not overlap, the inequality given in Eq. (2) indicates that the cross-spectral densities $S_{f_rf_s}(\omega)$ are zero. For this case, the cross-correlation $R_{y_1y_2}(t_1,t_2)$ between responses $y(x_1,t_1)$ and $y(x_2,t_2)$ becomes

$$R_{y_1y_2}(t_1,t_2) = \Phi(x_1)\Phi(x_2)\sum_{r=1}^{3} R_{q_rq_r}(t_1,t_2) \tag{33}$$

where

$$R_{q_rq_r}(t_1,t_2) = S_{f_rf_r}\int_{\omega_f - 0\cdot5\Delta\omega_f}^{\omega_f + 0\cdot5\Delta\omega_f} I_r(t_1,\omega)I_r^*(t_2,\omega)\,d\omega)/m^2 \tag{34}$$

The behavior of the autocovariance $R_{q_rq_r}(t) \equiv E[q_r^2(t)]$ is shown in Figs. 7–10 for various values of the parameters governing the response of the system. These parameters are the system's critical damping ratio ζ, the filter's center frequency ω_{f_r}, its bandwidth $\Delta\omega_{f_r} = 2\zeta\omega_{f_r}$, and its cutoff time t_r. The values of these parameters were chosen so as to bracket most cases of interest while at the same time providing an indication of trends in the response characteristics as a function of parameter variation.

These figures confirm the observation drawn from Figs. 2 and 3 in that the transient forced response to white noise passed through a narrow-band pass filter will overshoot its stationary value except when the filter's center frequency coincides with the system's natural frequency. They indicate that the transient

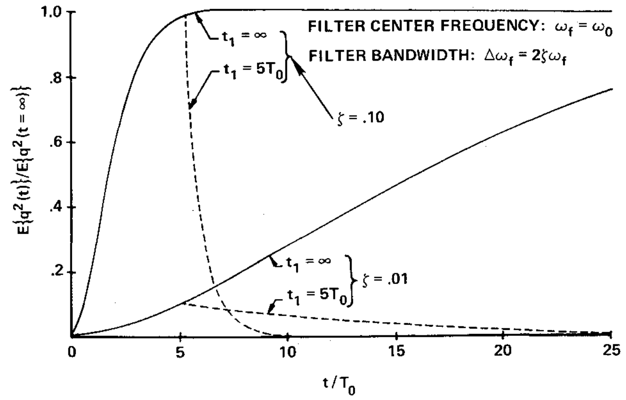


Fig. 7 **Mean square response to white noise passed through an idealized band pass filter for t_1 sec.**

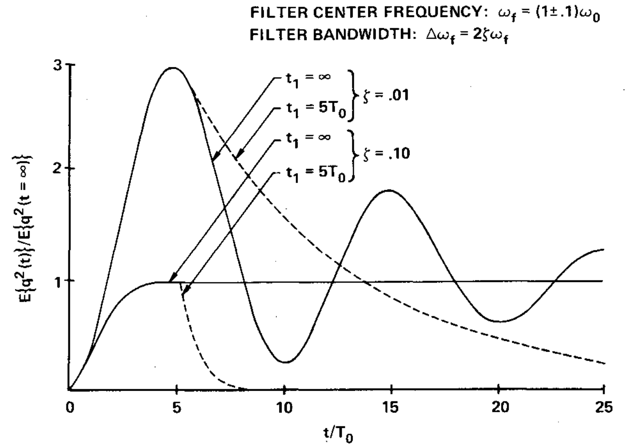


Fig. 8 **Mean square response to white noise passed through an idealized band pass filter for t_1 sec.**

forced response exhibits a damped periodic waveform whose shape and amplitude depend on the filter's center frequency and bandwidth. They also show that decreasing system damping tends to increase system response and the corresponding overshoot value.

Figures 9 and 10 indicate that the residual response is a decaying sinusoid having a frequency of $2\omega_0$, whose amplitude is strongly dependent on the time at which cutoff occurs, providing that t_r occurs prior to reaching stationariness. Furthermore, the residual response may exceed even the transient forced response as shown in Fig. 10. However, this phenomenon seems to occur only when the filter's center frequency is greater than the system's natural frequency.

Many of the response characteristics shown herein are consistent with Hasselman's work,[6,7] and both approaches show the importance of pulse duration, damping, and system natural frequency in evaluating the transient response.

The mean square response of the tip of the cantilever takes the form

$$E[y^2(l, t)] = \sum_{r=1}^{3} E[q_r^2(t)] \tag{35}$$

Its behavior is shown in Figs. 11a and 11b, normalized with respect to $E[q_2^2(t)]$, which is the response at $t = t_2$ due to the filter that is centered at the system's natural frequency. Since these mean square response curves significantly overshoot $E[q_2^2(t)]$, the transient forced response due to a filter centered at the system's natural frequency may be significantly increased by the residual reponse due to filters centered elsewhere.

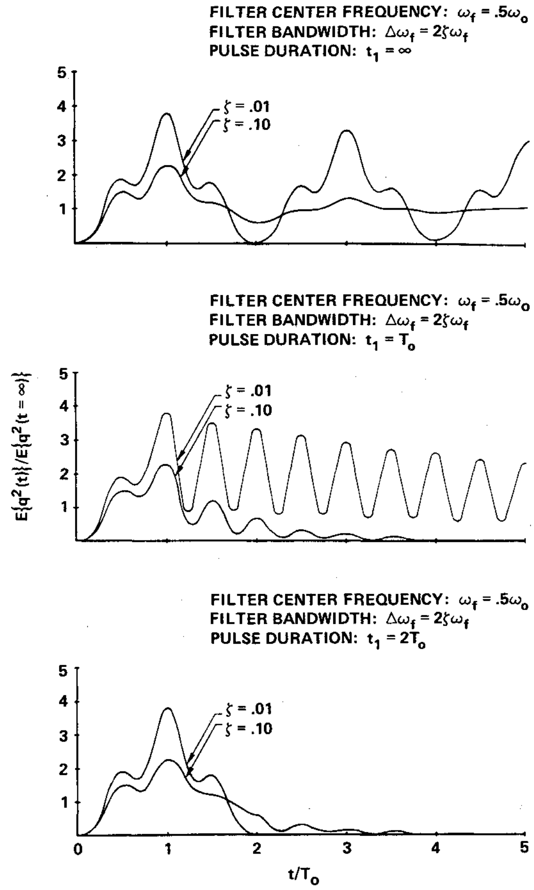


Fig. 9 **Mean square response to white noise passed through an idealized band pass filter for t_1 sec.**

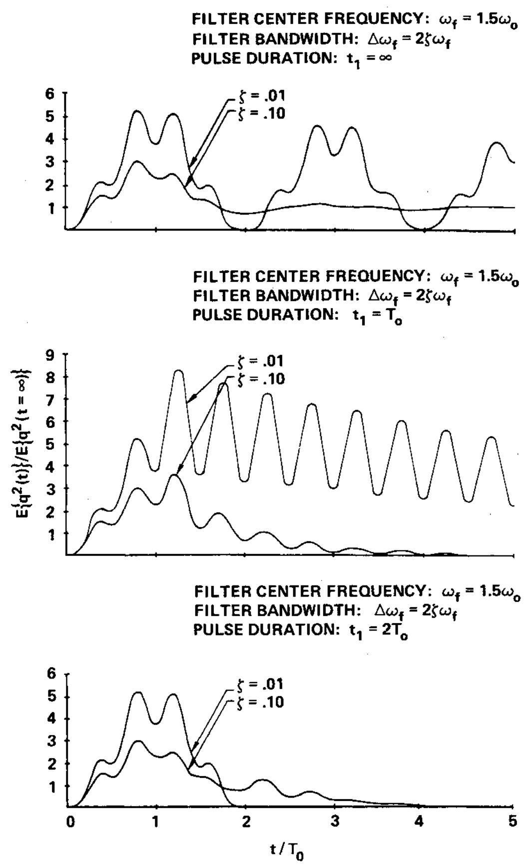
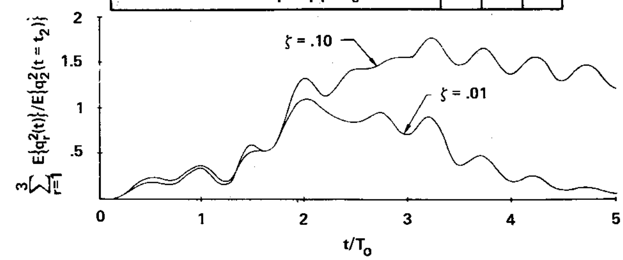


Fig. 10 **Mean square response to white noise passed through an idealized band pass filter for t_1 sec.**

SEGMENT r	1	2	3
FILTER CENTER FREQUENCY ω_{f_r}/ω_o	.5	1	1.5
FILTER BANDWIDTH $\Delta\omega_{f_r}/\omega_o$	ζ	2ζ	3ζ
PULSE DURATION $(t_r - t_{r-1})/T_o$	1	1	1



SEGMENT r	1	2	3
FILTER CENTER FREQUENCY ω_{f_r}/ω_o	.9	1	1.1
FILTER BANDWIDTH $\Delta\omega_{f_r}/\omega_o$	1.8ζ	2ζ	2.2ζ
PULSE DURATION $(t_r - t_{r-1})/T_o$	5	5	5

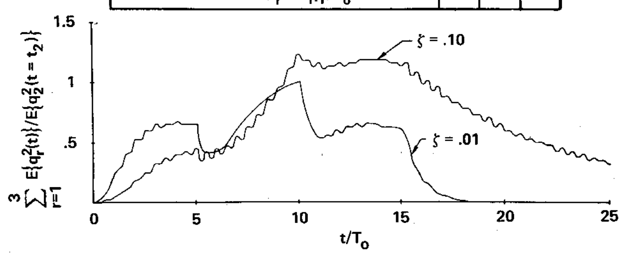


Fig. 11 Mean square response to white noise passed through three time-sequenced idealized band pass filters.

Conclusions

The mean square response of single-degree-of-freedom linear structural systems to segmented nonstationary random excitation has been derived in the form given by Eq. (3), where $S_{f_r f_s}(\omega)$ is the two-sided cross-spectral density function between covariance stationary zero-mean forcing functions $f_r(t)$ and $f_s(t)$, and $I_r(t, \omega)$ and $I_s(t, \omega)$ are the time-dependent frequency response functions for the corresponding deterministic shape functions $e_r(t)$ and $e_s(t)$.

The behavior of the product $I_r(t, \omega)I_s^*(t, \omega)$ for rectangular time-pulse modulation was studied in both the time and frequency domains as a function of pulse duration and system damping. It was found that the direct product terms $|I_r(t, \omega)|^2$ build up to 99% of their stationary value $|H(\omega)|^2$ in T_0/ζ sec, and die down to 1% of their stationary value in $T_0/2\zeta$ sec. The contribution of the cross-product terms $I_r(t, \omega)I_s^*(t, \omega)$ to the mean square response was found to range from nearly 50% (when the direct-spectral densities in adjacent segments were peaked at the system's natural frequency) to nearly zero (when the cross-spectral densities were fairly uniform or small over the frequency range $\omega_0 \pm 10\zeta\omega_0$).

The transient and stationary aspects of the mean square response of a uniform cantilever to stepped narrow-band base acceleration were studied as an example. It was found that 1) The transient response to white noise passed through a narrow-band-pass filter will overshoot its stationary value except when the filter's center frequency coincides with the system's damped natural frequency. The amount of overshoot is highly dependent upon the system's critical damping ratio and natural frequency, and the frequency content, duration, and intensity of the filter. 2) For stepped narrow-band white noise excitation, the transient forced response due to a filter centered at the system's damped natural frequency may be significantly increased by the residual response due to filters centered elsewhere.

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